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QUANTUM (2,2) SUPERGRAVITY *

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ABSTRACT

We describe various aspects of two-dimensional $N = 2$ supergravity in superspace. We present the solution to the constraints in terms of unconstrained prepotentials, and the different superspace measures (full and chiral) used in the construction of invariant actions. We discuss aspects of the theory in light-cone gauge, including the Ward identities for correlation functions defined with respect to the induced supergravity action.

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1 Introduction

Three years ago, P. van Nieuwenhuizen and one of the authors (M.T.G) performed some loop calculations in Polyakov's quantum induced gravity in light-cone gauge [1]. We recall that away from the critical dimension, the induced gravity action takes the form

$$\mathcal{S}_{ind} = -\frac{c}{24\pi}\int d^2x R\frac{1}{\Box}R \tag{1.1}$$

which in light-cone gauge becomes

$$\begin{aligned}\mathcal{S}_{ind} &= -\frac{c}{24\pi}\int d^2x\partial_-^2 h_{++}\frac{1}{1-\frac{1}{\partial_+}h_{++}\partial_-}\frac{\partial_-}{\partial_+}h_{++} \\ &= -\frac{c}{24\pi}\int d^2x\left[h\frac{\partial_-^3}{\partial_+}h - h\left(\frac{\partial_-^2}{\partial_+}h\right)^2 + \cdots\right] \quad .\end{aligned}$$

To determine the effective action, $\mathcal{S}_{eff}$, one computes perturbatively in powers of $1/c$. The one-loop results are given in [2, 3]. In particular one finds renormalization $c \to Z_c c$ and $h \to Z_h h$, consistent with results in the equivalent KPZ or WZNW formulation [4], where one proves that under quantum corrections

$$\frac{c}{6} \to k+2 = \frac{1}{12}[c-1-\sqrt{(c-1)(c-25)}] \tag{1.2}$$

As suggested by the equivalence, as well as other arguments [5, 6], one expects a stronger result, namely the relationship between the effective and induced actions is given by

$$\mathcal{S}_{eff}(h,c) = \mathcal{S}_{ind}(Z_h h, Z_c c) \quad . \tag{1.3}$$

The work presented in [3] was done in part to understand perturbatively what happens to c and to h under renormalization (which is understood differently in the KPZ or WZNW formulation), and in part to gain some experience with quantum calculations in non-local field theories. The calculations turned out to exhibit some unpleasant features: first, there were ultraviolet linearly divergent loop integrals, so that results were in principle routing dependent. Second, and perhaps related to this first problem, it appeared that the results were regularization dependent. The work used the Polyakov exponential cutoff regularization, but calculations by other authors [7], using Pauli-Villars regularization, led to somewhat different results. At any rate, going beyond one loop in order to check the relationship between S_{eff} and S_{ind} appeared prohibitively difficult.

It is conceivable that the situation might be better in supergravity, where the ultraviolet behaviour is generally much improved, and supergraph techniques may simplify higher-loop calculations. It turns out that the ultraviolet behaviour is not

sufficiently improved in the case of (1,0) or (1,1) supergravity [8], but is for (2,0) [9] or (2,2) supergravity. (However, there are unpleasant features of (2,0) supergravity, such as anomalies induced by the lack of left-right symmetry which make it a less appealing candidate for further investigation). There are other aspects of Polyakov's induced gravity which one may wish to generalize to the (2,2) case. Foremost are the Ward identities which exhibit the hidden $SL(2,C)$ symmetry of induced gravity and in principle solve completely the theory. These Ward identities have also been used [10] to determine the dressing of nonconformal matter one-loop β-functions by induced gravity, $\beta_0 \to \beta = \frac{k+2}{k+1}\beta_0$, and a similar approach would be interesting in the (2,2) supergravity case where perturbative methods indicate the absence of such corrections [11]. Finally the (2,2) case is interesting in its own right because one believes that Polyakov's light-cone results can be mapped onto the WZNW model, but for the (2,2) case there are apparently problems constructing the WZNW action.

In order to study these questions, which are of a quantum nature, one needs to have a formulation of $N = 2$ supergravity in terms of unconstrained prepotentials which allows functional integration, etc.. Such a formulation has been absent in the past. Recently, we have developed a prepotential approach to $N = 2$ supergravity which allows discussion of the quantum properties of the theory, its light-cone formulation, and the derivation of the Ward identities. In the following we summarize our procedure and results. Further details can be found in the published references.

2 Solution of (2,2) Constraints

From the two-dimensional $N = 2$ nonminimal $U_A(1) \otimes U_V(1)$ supergravity theory, two versions of minimal (2,2) supergravity can be obtained [12, 13] – the axial $U_A(1)$ version and the vector $U_V(1)$ version – depending upon which tangent space symmetry one gauges. We focus on the axial version, which is related to minimal four-dimensional $N = 1$ supergravity by dimensional reduction. (Results for the $U_V(1)$ case can be easily obtained from the axial solution.)

We work in a spinor light-cone basis and use superspace coordinates ($x^{\ddagger}$, $x^{=}$; θ^+, θ^-, $\theta^{\dot+}$, $\theta^{\dot-}$). In the nonminimal theory the spinorial covariant derivatives are

$$\nabla_\alpha \quad = \quad E_\alpha + \Phi_\alpha \mathcal{M} + \Sigma'_\alpha \mathcal{Y}' + \Sigma_\alpha \mathcal{Y} \qquad (2.1)$$

where $\alpha = \pm$, with similar expressions for the complex conjugates, as well as the vectorial derivatives. The tangent space Lorentz, $U_V(1)$ and $U_A(1)$ generators are $\mathcal{M}$, $\mathcal{Y}$ and $\mathcal{Y}'$, with the associated connections Φ_α, Σ_α and Σ'_α. The vielbein is given by $E_A = E_A{}^M D_M$.

We list the constraints for the $U_A(1) \otimes U_V(1)$ theory [12, 13]. They are

$$\{\nabla_+, \nabla_+\} \quad = \quad 0 \qquad , \qquad \{\nabla_-, \nabla_-\} = 0$$

$$\{\nabla_+, \nabla_{\dot{+}}\} \;=\; i\nabla_{\ddagger} \quad , \quad \{\nabla_-, \nabla_{\dot{-}}\} = i\nabla_{=} \quad , \tag{2.2}$$

as well as

$$\begin{aligned} \{\nabla_+, \nabla_-\} &= -\frac{1}{2}\bar{R}(\mathcal{M} - i\mathcal{Y}') \\ \{\nabla_+, \nabla_{\dot{-}}\} &= -\frac{1}{2}\bar{F}(\mathcal{M} - i\mathcal{Y}) \end{aligned} \tag{2.3}$$

and their complex conjugates.

To find a description of one of the minimal $U(1)$ theories, we follow a procedure similar to that used in the four dimensional case. We start with the *full* $U_A(1)\otimes U_V(1)$ gauge group, and solve the constraints for the nonminimal theory. The solution is derived in terms of two prepotentials, a real vector superfield H^m and a general complex superfield compensator S. We obtain the solution in terms of the "hat" differential operators

$$\hat{E}_\pm = e^{-H} D_\pm e^{H} \quad , \quad H = H^m i\partial_m \quad , \tag{2.4}$$

with the spinorial vielbein given by

$$\begin{aligned} E_+ &\equiv e^{\bar{S}}(\hat{E}_+ + A_+{}^-\hat{E}_-) \\ E_- &\equiv e^{\bar{S}}(\hat{E}_- + A_-{}^+\hat{E}_+) \end{aligned} \tag{2.5}$$

(again with corresponding expressions for the complex conjugates). The quantities $A_\alpha{}^\beta$ in (2.5) can be solved for and are given explicitly in terms of H^m *only*, as is the vielbein determinant E, whereas the connections Ω_α, Γ_α and Σ_α are functions of H^m, as well as S and $\bar{S}$ [13].

Now that we have these equations, we reduce the theory to one of its minimal forms by "degauging", i.e. by eliminating one of the (extra) $U(1)$ symmetries. To obtain the minimal $U_A(1)$ ($U_V(1)$) theory, we restrict the gauge group by setting $\Sigma_\alpha = 0$ ($\Sigma'_\alpha = 0$), or equivalently the field strength $F = 0$ ($R = 0$). Imposing this additional restriction in the $U_A(1)$ case [13], we find that the superfield $\bar{S}$ can be expressed in terms of an arbitrary covariantly antichiral superfield $\bar{\sigma}$ as

$$e^{\bar{S}} = e^{\bar{\sigma}} \frac{\left[1 \cdot e^{-\overleftarrow{H}}\right]^{-\frac{1}{2}}}{[1 - A_+{}^- A_-{}^+]^{\frac{1}{2}}} E^{-\frac{1}{2}} \quad . \tag{2.6}$$

(Here $\overleftarrow{H}$ indicates that the differential operator in $H^m i\partial_m$ acts on objects to its left.) This completes the degauging to the minimal $U_A(1)$ theory.

The unconstrained real vector superfield H^m and the chiral scalar superfield σ are the prepotentials of minimal $U_A(1)$ (2,2) supergravity. To obtain the mirror image

$U_V(1)$ theory, one simply interchanges $-$ and $\dot{-}$ everywhere (as well as interchanging R with F, and $\mathcal{Y}$ with $\mathcal{Y}'$). This amounts to replacing the chiral superfield σ with a twisted chiral superfield $\tilde{\sigma}$.

With this solution at hand we can now discuss invariant actions. Details can be found in [14], and we simply list the results here. Since the vielbein determinant depends solely on H^m, it is clear that we can construct invariant actions in *full* superspace that are independent of the compensator σ, such as

$$\mathcal{S} = \int d^2x d^4\theta E^{-1} \mathcal{L} \tag{2.7}$$

where $\mathcal{L}$ is an arbitrary scalar function of superfields.

For the minimal $U_A(1)$ theory we can rewrite the full superspace integral above as as integral over chiral superspace,

$$\mathcal{S} = \int d^2x d^2\theta \mathcal{E}^{-1} \bar{\nabla}^2 \mathcal{L}|_{\bar{\theta}=0} \tag{2.8}$$

with the chiral measure

$$\mathcal{E}^{-1} = e^{-2\sigma}(1.e^{\overleftarrow{H}}) \quad . \tag{2.9}$$

The case for the $U_V(1)$ theory is similar. We can rewrite the full superspace integral as

$$\mathcal{S} = -\int d^2x d\theta^+ d\theta^{\dot{-}} \tilde{\mathcal{E}}^{-1} \nabla_{\dot{+}} \nabla_{-} \mathcal{L}|_{\tilde{\theta}=0} \tag{2.10}$$

with the twisted chiral measure

$$\tilde{\mathcal{E}}^{-1} = e^{-2\tilde{\sigma}}(1.e^{\overleftarrow{H}}) \quad . \tag{2.11}$$

These measures can be used for arbitrary (twisted) chiral integrands.

3 Transformation Laws

We now examine the gauge transformations of the prepotentials in this description of (2,2) supergravity.

In the solution of the constraints knowledge of the result in the four-dimensional case [15] was used as a guide in eliminating many irrelevant superfields through algebraic gauge transformations to specific supersymmetric gauges. The desired form of $H = H^m i\partial_m$ (H^m real and vectorial) was determined by implicitly using the usual K invariance of the covariant derivatives ($\nabla'_A = e^{iK}\nabla_A e^{-iK}$, K a real scalar superfield), as well as some of the Λ invariance which appears as a result of the

solution. Specifically, it was possible to gauge away the imaginary part of H^m, and spinorial superfields H^α, $H^{\dot\alpha}$. At this point some Λ invariance still remains however, and we now discuss this, influenced again by experience with the four-dimensional $N = 1$ situation.

We start by examining the invariance under superspace coordinate transformations of the kinetic action for covariantly chiral and antichiral superfields [16]. Covariantly chiral and antichiral scalar superfields Φ, $\bar\Phi$ are defined by $\nabla_{\dot\pm}\Phi = \nabla_\pm\bar\Phi = 0$. They can be expressed in terms of ordinary chiral and antichiral superfields ϕ, $\bar\phi$, by

$$\Phi = e^H \phi e^{-H} \quad , \quad \bar\Phi = e^{-H}\bar\phi e^H \quad . \tag{3.1}$$

Their kinetic action is

$$\begin{aligned} S &= \int d^2x d^4\theta E^{-1}\bar\Phi\Phi \\ &= \int d^2x d^4\theta E^{-1}\left(e^{-H}\bar\phi\right)e^{-H}\left(e^{2H}\phi\right) \\ &= \int d^2x d^4\theta E^{-1} e^{\overleftarrow{H}}\left(\bar\phi e^{2H}\phi\right) \quad . \end{aligned}$$

where we have performed some integration by parts.

Ordinary chiral and antichiral superfields transform under supercoordinate transformations as

$$\phi \to e^{i\Lambda}\phi \quad , \quad \bar\phi \to e^{i\bar\Lambda}\bar\phi \tag{3.2}$$

where Λ and $\bar\Lambda$ are given by

$$\Lambda = \Lambda^m i\partial_m + \Lambda^\alpha iD_\alpha + \Lambda^{\dot\alpha} iD_{\dot\alpha} \quad , \quad \bar\Lambda = \bar\Lambda^m i\partial_m + \bar\Lambda^\alpha iD_\alpha + \bar\Lambda^{\dot\alpha} iD_{\dot\alpha} \quad . \tag{3.3}$$

The above action will be invariant under these transformations provided that

$$e^{2H} \to e^{i\bar\Lambda}e^{2H}e^{-i\Lambda} \quad , \quad E^{-1}e^{\overleftarrow{H}} \to E^{-1}e^{\overleftarrow{H}}e^{i\overleftarrow{\bar\Lambda}} \quad . \tag{3.4}$$

Similarly, by requiring invariance of the chiral integral in (2.8) we find

$$e^{-2\sigma}(1.e^{\overleftarrow{H}}) \to e^{-2\sigma}(1.e^{\overleftarrow{H}})e^{i\overleftarrow{\Lambda}} \tag{3.5}$$

The Λ's are restricted by two requirements: they must be (anti)chirality-preserving, i.e. $D_\pm e^{i\bar\Lambda}\bar\phi = 0$ and they must maintain the vector nature of the operator $H = H^m i\partial_m$. These conditions imply that the Λ's can be expressed in terms of arbitrary spinor parameters L^α, $L^{\dot\alpha}$. The precise relation is given in [16]. In particular, at the linearized level, the transformations of the prepotentials are

$$\delta H^{\ddagger} = \frac{i}{2}(D_{\dot -}L^+ - D_- L^{\dot +})$$

$$
\begin{aligned}
\delta H^{=} &= \frac{i}{2}(D_{\dot{+}}L^{-} - D_{+}L^{\dot{-}}) \\
\delta\sigma &= -\frac{i}{2}\bar{D}^2(D_{+}L^{+} - D_{-}L^{-}) \\
\delta\bar{\sigma} &= -\frac{i}{2}D^2(D_{\dot{+}}L^{\dot{+}} - D_{\dot{-}}L^{\dot{-}}) \quad .
\end{aligned} \tag{3.6}
$$

4 Reaching Light-Cone Gauge

Going to a specific gauge, where certain components of gauge fields $\mathcal{V}$ are set to zero, involves examining their gauge transformations and showing that for any such transformation, $\mathcal{V} \to \mathcal{V} + \delta\mathcal{V} = \mathcal{V} + \mathcal{D}\mathcal{L}$, one can solve for the gauge parameter $\mathcal{L}$ for any $\delta\mathcal{V}$. We go to light-cone gauge by choosing $x^{=}$ as "time", so that $1/\partial_{\ddagger}$ is local and can be used when solving for gauge parameters without introducing propagating ghosts. We show that in H^m it is possible to gauge away all of $H^{=}$ by using the gauge parameters L^{-} and $L^{\dot{-}}$, and the compensators σ, $\bar{\sigma}$, and certain components of $H^{\ddagger}$ by using L^{+} and $L^{\dot{+}}$. It is, of course, sufficient to examine the linearized transformations.

We consider first the transformation

$$
\delta H^{=} = \frac{i}{2}(D_{\dot{+}}L^{-} - D_{+}L^{\dot{-}}) \quad . \tag{4.1}
$$

Component by component, $H^{=}$ can be gauged away completely by components of L^{-} or $L^{\dot{-}}$, with nothing left of L^{-} to use in $\delta\sigma \sim \bar{D}^2 D_{-}L^{-}$ to gauge away the compensator [16].

We then look at the gauge transformations induced by L^{+} and $L^{\dot{+}}$. Using them to gauge away all of $H^{\ddagger}$ takes us to superconformal gauge; instead we use them to gauge away the compensators, and some of the lower components of $H^{\ddagger}$, by a Wess-Zumino gauge choice which eliminates some left-over gauge invariance. We find thus a gauge in which $\sigma = 0$ and the prepotential has the form

$$
\begin{aligned}
H^{\ddagger} = {} & \theta^{-}\theta^{\dot{-}}[h_{=}{}^{\ddagger} + \theta^{+}\psi_{\dot{-}}{}^{\ddagger} - \theta^{\dot{+}}\psi_{-}{}^{\ddagger} - \theta^{+}\theta^{\dot{+}}D^{\ddagger}] \\
& + \theta^{-}[e^{\frac{i}{2}\theta^{+}\theta^{\dot{+}}\partial_{\ddagger}}(\lambda_{-}^{\ddagger} + \theta^{+}N^{\ddagger})] + \theta^{\dot{-}}[e^{-\frac{i}{2}\theta^{+}\theta^{\dot{+}}\partial_{\ddagger}}(\lambda_{\dot{-}}^{\ddagger} + \theta^{\dot{+}}\bar{N}^{\ddagger})]
\end{aligned} \tag{4.2}
$$

Therefore, in light-cone gauge $H^{\ddagger}$ has a decomposition in terms of $(2,0)$ superfields, one of them real, the other two chiral and antichiral, with respect to $D_{\dot{+}}$ and D_{+}. Absorbing the explicit θ^{-} and $\theta^{\dot{-}}$ into the definition of these superfields, we write

$$
H^{\ddagger} \equiv \mathcal{H}^{\ddagger} + \chi^{\ddagger} + \bar{\chi}^{\ddagger} \quad , \tag{4.3}
$$

$$
\mathcal{H}^{\ddagger} = \theta^{-}\theta^{\dot{-}}\mathcal{H}_{=}{}^{\ddagger} \quad , \quad \chi^{\ddagger} = \theta^{-}\chi_{-}{}^{\ddagger} \quad , \quad \bar{\chi}^{\ddagger} = \theta^{\dot{-}}\chi_{\dot{-}}{}^{\ddagger} \quad . \tag{4.4}
$$

We now list, in light-cone gauge, the expressions for the various relevant quantities in the theory. We find it convenient to describe the results in terms of the field

$$\tilde{\mathcal{H}}^{\ddagger} = \mathcal{H}^{\ddagger} + \frac{i}{2}\chi^{\ddagger} \overleftrightarrow{\partial}_{\ddagger} \bar{\chi}^{\ddagger} \quad , \tag{4.5}$$

and note that the dependence of the geometrical quantities on the prepotentials is almost linear in this gauge:

$$E^{-1} = 1 - [D_{\dot{+}}, D_+]\tilde{\mathcal{H}}^{\ddagger} - i\partial_{\ddagger}(\chi - \bar{\chi})^{\ddagger} + 2\partial_{\ddagger}\chi^{\ddagger}\partial_{\ddagger}\bar{\chi}^{\ddagger} + iD_+\chi^{\ddagger} \overleftrightarrow{\partial}_{\ddagger} D_{\dot{+}}\bar{\chi}^{\ddagger} \tag{4.6}$$

$$\begin{aligned} e^{2\bar{S}} &= 1 + 2D_+D_{\dot{+}}\tilde{\mathcal{H}}^{\ddagger} + 2i\partial_{\ddagger}\bar{\chi}^{\ddagger} + \chi^{\ddagger}\partial_{\ddagger}^2\bar{\chi}^{\ddagger} + \partial_{\ddagger}\chi^{\ddagger}\partial_{\ddagger}\bar{\chi}^{\ddagger} + iD_+\chi^{\ddagger}\overleftrightarrow{\partial}_{\ddagger}D_{\dot{+}}\bar{\chi}^{\ddagger} & (4.7)\\ e^{2S} &= 1 - 2D_{\dot{+}}D_+\tilde{\mathcal{H}}^{\ddagger} - 2i\partial_{\ddagger}\chi^{\ddagger} + \bar{\chi}^{\ddagger}\partial_{\ddagger}^2\chi^{\ddagger} + \partial_{\ddagger}\chi^{\ddagger}\partial_{\ddagger}\bar{\chi}^{\ddagger} + iD_+\chi^{\ddagger}\overleftrightarrow{\partial}_{\ddagger}D_{\dot{+}}\bar{\chi}^{\ddagger} & (4.8)\end{aligned}$$

$$\begin{aligned} \bar{R} &= -2e^{-i(\chi+\tilde{\mathcal{H}})\partial}D^2[2i\partial_{\ddagger}(\tilde{\mathcal{H}}+\chi)^{\ddagger} - 4\partial_{\ddagger}\chi^{\ddagger}\partial_{\ddagger}\bar{\chi}^{\ddagger} - \chi^{\ddagger}\partial_{\ddagger}^2\bar{\chi}^{\ddagger} - \bar{\chi}^{\ddagger}\partial_{\ddagger}^2\chi^{\ddagger}] & (4.9)\\ R &= 2e^{i(\bar{\chi}+\tilde{\mathcal{H}})\partial}\bar{D}^2[-2i\partial_{\ddagger}(\tilde{\mathcal{H}}+\bar{\chi})^{\ddagger} - 4\partial_{\ddagger}\chi^{\ddagger}\partial_{\ddagger}\bar{\chi}^{\ddagger} - \chi^{\ddagger}\partial_{\ddagger}^2\bar{\chi}^{\ddagger} - \bar{\chi}^{\ddagger}\partial_{\ddagger}^2\chi^{\ddagger}] \quad . & (4.10)\end{aligned}$$

5 Light-cone gauge transformations

Ultimately we want to derive Ward identities for correlation functions defined by (functional) averaging with the (nonlocal) induced supergravity action. They are obtained from the invariance of the functional integral under a change of variables which is a field transformation. The only requirement is that the variation of the induced action be local, which can be achieved if the field transformation is a gauge transformation for which the induced action is anomalous. In our case this is true for the general gauge transformations of the prepotentials, when restricted to light-cone gauge and chosen to preserve the form of $H^{\ddagger}$.

We consider the general gauge transformation

$$e^{2H} \rightarrow e^{i\bar{\Lambda}}e^{2H}e^{-i\Lambda} \tag{5.1}$$

choose $L^- = L^{\dot{-}} = 0$, and suitably restrict L^+ and $L^{\dot{+}}$. This procedure is rather involved (details are given in [16]) and we simply state the result here. We find that the final form of the residual light-cone transformations is, with arbitrary parameters $\alpha^{\ddagger}$, $\gamma^{\ddagger}_{\dot{-}}$ and $\bar{\gamma}^{\ddagger}_{-}$,

$$\begin{aligned} \delta\chi^{\ddagger} &= iD_{\dot{+}}[\gamma^{\ddagger}_{\dot{-}} + \frac{1}{2}\chi^{\ddagger}D_+\alpha^{\ddagger} - \frac{1}{2}\alpha^{\ddagger}D_+\chi^{\ddagger}] \\ \delta\bar{\chi}^{\ddagger} &= -iD_+[\bar{\gamma}^{\ddagger}_{-} + \frac{1}{2}\alpha^{\ddagger}D_{\dot{+}}\bar{\chi}^{\ddagger} - \frac{1}{2}\bar{\chi}^{\ddagger}D_{\dot{+}}\alpha^{\ddagger}] \quad , \end{aligned} \tag{5.2}$$

and also that a useful quantity which transforms simply is

$$\check{\mathcal{H}}^{\ddagger} = \mathcal{H}^{\ddagger} + i\chi^{\ddagger} \overleftrightarrow{\partial}_{\ddagger} \bar{\chi}^{\ddagger} \quad , \tag{5.3}$$

for which we have

$$\delta\check{\mathcal{H}}^{\ddagger} = -\frac{1}{2}\theta^{-}\theta^{\dot{-}}\partial_{=}\alpha^{\ddagger} + \alpha^{\ddagger}\partial_{\ddagger}\check{\mathcal{H}}^{\ddagger} - iD_{\dot{+}}\alpha^{\ddagger}D_{+}\check{\mathcal{H}}^{\ddagger} - iD_{+}\alpha^{\ddagger}D_{\dot{+}}\check{\mathcal{H}}^{\ddagger} - \check{\mathcal{H}}^{\ddagger}\partial_{\ddagger}\alpha^{\ddagger} \quad . \tag{5.4}$$

6 Light-cone gauge Ward identities

The induced $(2,2)$ supergravity action is given by

$$S_{ind} = \frac{c}{4\pi}\int d^6 z \bar{R}\frac{1}{\Box_c}R \tag{6.1}$$

where $\Box_c$ is the supergravity covariantized d'Alembertian. It is obtained by coupling superconformal matter to supergravity. Integrating out the matter gives rise, because of the superconformal anomaly, to this induced action.

The correlation functions in the presence of the induced action are defined by

$$< X(z_1, z_2, ...z_n) >= \int \mathcal{D}(H,\phi)e^{S_{ind}(H)+S_m(\phi)}X(z_1, z_2, ...z_n) \quad , \tag{6.2}$$

where $X(z_1, z_2, ...z_n)$ denotes a product of supergravity or matter fields, and S_m is the action for matter. In the functional integral we make a change of integration variables which is the residual Λ-transformation defined in the previous section, and assume that S_m is invariant, while the induced action varies into the (local) anomaly. We obtain the Ward identity

$$\begin{aligned} 0 &= \int \mathcal{D}(H,\phi)e^{S_{ind}(H)+S_m(\phi)}\left[\delta S_{ind}X(z_1, z_2, ...z_n) + \sum_i \delta_i X(z_1, z_2, ...z_n)\right] \\ &= < \delta S_{ind}X(z_1, z_2, ...z_n) > + \sum_i < \delta_i X(z_1, z_2, ...z_n) > \quad , \end{aligned} \tag{6.3}$$

where $\delta_i X(z_1, z_2, ...z_n)$ is the variation of the i'th field in $X(z_1, z_2, ...z_n)$. The functional integration in light-cone gauge is over $\chi^{\ddagger}$, $\bar{\chi}^{\ddagger}$ and $\mathcal{H}^{\ddagger}$, with the gauge transformations given in (5.2) and (5.4). For the matter fields, e.g. chiral scalar superfields with weight λ, we assume transformations such as $\delta\phi = i[\Lambda, \phi] + i\lambda(1.\overleftarrow{\Lambda})\phi$, i.e. $\delta\phi = -i\bar{D}^2(L^{+}D_{+}\phi) + i\lambda(\bar{D}^2 D_{+}L^{+})\phi$.

The variation of the induced action is

$$\begin{aligned} \delta S_{ind} &= \frac{ic}{\pi}\int d^2x d^4\theta \left\{\left(i[D_{+}, D_{\dot{+}}]\partial_{\ddagger}\check{\mathcal{H}}^{\ddagger} + \partial^2_{\ddagger}(\chi^{\ddagger} - \bar{\chi}^{\ddagger})\right)\alpha^{\ddagger} \right. \\ &\qquad \left. + 2\partial^2_{\ddagger}\chi^{\ddagger}D_{+}\bar{\gamma}^{\ddagger}_{\dot{-}} - 2\partial^2_{\ddagger}\bar{\chi}^{\ddagger}D_{\dot{+}}\gamma^{\ddagger}_{-}\right\} \quad , \end{aligned} \tag{6.4}$$

and we note that α and γ are arbitrary gauge parameters.

We can verify, as an explicit example in perturbation theory, the Ward identity for the correlator $<\bar{\phi}(y)\phi(z)>$ for an ordinary chiral scalar superfield ($\lambda = 0$). The matter action is

$$
\begin{aligned}
S_m &= \int d^6 z E^{-1} \left(e^{-H}\bar{\phi}\right)\left(e^{H}\phi\right) \\
&= \int d^6 z (\bar{\phi}\phi - 2H^{\ddagger} D_{\dot{+}}\bar{\phi} D_{+}\phi + \cdots) \quad .
\end{aligned} \tag{6.5}
$$

The Ward identity is

$$
<\delta S_{ind}\bar{\phi}\phi> + <\delta\bar{\phi}\phi> + <\bar{\phi}\delta\phi> = 0 \tag{6.6}
$$

with

$$
\begin{aligned}
\delta\phi &= -i\bar{D}^2[\theta^{\dot{-}}(\alpha^{\ddagger} - 2i\chi^{\ddagger}\partial_{\ddagger}\alpha^{\ddagger} + D_{\dot{+}}D_{+}(\chi^{\ddagger}\alpha^{\ddagger}) + 2D_{\dot{+}}\gamma^{\ddagger}_{\dot{-}})D_{+}\phi] \\
\delta\bar{\phi} &= -iD^2[\theta^{-}(\alpha^{\ddagger} + 2i\bar{\chi}^{\ddagger}\partial_{\ddagger}\alpha^{\ddagger} - D_{+}D_{\dot{+}}(\bar{\chi}^{\ddagger}\alpha^{\ddagger}) + 2D_{+}\bar{\gamma}^{\ddagger}_{-})D_{\dot{+}}\bar{\phi}] \quad .
\end{aligned} \tag{6.7}
$$

We note that terms linear in c trivially satisfy the Ward identity, terms independent of c lead to tree graphs and terms proportional to $1/c^L$ give rise to loops. Obviously the terms that depend on α and γ must separately satisfy the Ward identity. We consider the α-dependent part and obtain

$$
\begin{aligned}
&\frac{ic}{\pi}\int d^2x d^4\theta < i[D_{+}, D_{\dot{+}}]\partial_{\ddagger}\mathcal{H}^{\ddagger}(x)\alpha^{\ddagger}(x)\bar{\phi}(y)\phi(z)> \\
&\quad -i < D^2[\theta^{-}\alpha^{\ddagger}D_{\dot{+}}\bar{\phi}(y)]\phi(z)> -i < \bar{\phi}(y)\bar{D}^2[\theta^{\dot{-}}\alpha^{\ddagger}D_{+}\phi(z)]> = 0
\end{aligned} \tag{6.8}
$$

which we have verified by an explicit tree-level calculation in [16].

7 Comments

We have outlined the description of (2,2) supergravity in terms of unconstrained prepotentials, and its light-cone properties. We note that the choice of a *ghost-free* light-cone gauge, with no residual gauge invariance for the induced action, necessarily forces us to a Wess-Zumino-type gauge, which does not have manifest (2,2) supersymmetry. In this respect the situation is different from the case of the other supergravity theories that have been studied. However, the lack of manifest (2,2) supersymmetry is not a significant impediment to most applications.

The primary application of the light-cone formulation of (2,2) supergravity is the investigation of properties of the induced non-local action S_{ind}. Specifically, one can generalize to the (2,2) case Polyakov's $SL(2,C)$ symmetry for induced gravity,

as was done for (1,0) and (1,1) supergravity in [8] and for (2,0) supergravity in [9]. Comparison of our results with those of this last reference, and in particular the transformation in (5.4), shows that our Ward identities involving $\mathcal{H}^{\ddagger}$ are the same as those in the (2,0) case so that, upon solving them, one expects to find the same associated $OSp(2|2)$ current algebra. However, in the (2,2) theory which involves the superfields χ, $\bar{\chi}$ in addition to $\mathcal{H}$, one avoids the anomalies due to the inherent left-right asymmetry of the (2,0) theory.

The bosonic Ward identities have been used to determine the induced gravity corrections to the β-functions in non-conformal theories [17, 10, 18]. One of the authors (M.T.G.), with D. Zanon [11], has examined supergravitational dressing in perturbation theory. Corrections to the $N = 1$ σ-model β-functions were found, but there were no one-loop corrections in the $N = 2$ case. It is of interest to recover these results by using the Ward identities we have derived.

Lastly, one can attempt to do higher loop calculations in the induced supergravity in light-cone gauge. At one-loop, one gets a result consistent with $c - 1 = 2(k + 1)$ [19]. As mentioned in the introduction, higher loop calculations generalizing those of [3] may be feasible, and free of the problems encountered in the bosonic case.

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